Formalizing and solving some elementary problems

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Abstract: : In an IT system, a state is generally described by means of a cluster of state variables. The values assigned to these variables will allow the defining, in a univocal manner, a given state. Generally speaking, these variables describe the attributes of different objects (or facts, situations etc.) of the problem environment. In order to achieve the final sate, the solving process must have a series of transformation operators, which will allow a progressive transition from the initial state towards the final state. These operators are also called transition operators and are seen as a way of reducing differences between the initial state and the final one.

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1. Introduction

We have discussed the formalizing role and importance in[1]. There we have also presented a classification of problems and saw that directly solvable problems are those in the category of well defined problems.

The present paper deals with solving elementary problems which are part of the transformation problem category.

For problems in the transformation category, we can have the following aspects.

According to[2], solving such a problem implies the progress from one state to another, starting from the initial state, or more, towards the final solution (final state, or more). This is made up of the cluster of possible states of that problem.

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In order to achieve the final sate, the solving process must have a series of transformation operators, which will allow a progressive transition from the initial state

[1] Giurgiteanu N., Buligiu I., Formalizarea şi rezolvarea problemelor, Timisoara Journal of Economics (forthcoming), 2010
[2] Newell A., Simon H. A., "Human problem solving, Englewood Cliffs", N. J., Erlbaum., 1972

towards the final state. These operators are also called transition operators and are seen as a way of reducing differences between the initial state and the final one.

Thus, solving a problem implies the succession of the following three stages:

Formal distribution of the problem states (the cluster of state variables) - in this stage one can make use of formalism in representing knowledge, or directly, in the case of structural data, such as lists or tables;

Formal representation of transactional operators – here a logical formalism may be used.

Choosing the solving algorithm, which consists of choosing the operators to be applied, as well as establishing on which sates of the problem these operators shall be applied with the aim of finding the final state (the aim state).

As a direct application of those stated above, let's assume the following problem needs to be solved: A peasant owns a wolf, a goat and a cabbage (he can hardly prevent the wolf from eating the goat, and the goat from eating the cabbage). When coming near a river, the peasant wants to cross over with his small fortune, by means of a boat in which only himself and one of his three possessions has room. How will he achieve that? ur goal is to simulate as closely as possible, this sample.

2. CLASSIFYING THE PROBLEM

It is obvious that this problem belongs to the transformation category, that is there is the initial state of the problem: the peasant is on the bank of the river with his boat and his small fortune. The final state he has to achieve is to reach the other bank, together with all his possessions.

The attempt to formalize the problem solution justifies our introduction of the following notations:

C, to represent the cabbage;
W, to represent the wolf;
G, to represent the goat;
H, to represent the human and
B, to represent the boat;
Lb, to represent the left bank;
Rb, to represent the right bank.

The space of the problem states. To begin with, we sall emphasize the space of the problem states. Thus, we shall represent each state of situations in which we may find ourselves, throughout solving the problem, by means of two variables materializing each state of the two banks of the river. This shall be achieved by means of two lists, one for each bank, each list being made up of 5 elements: the first element is the human, the second is the cabbage, the third is the wolf, and the last two are the goat, respectively the boat.

Assuming the peasant, his possessions and his boat are on the left bank, then Lb, the state of the left bank has the form Lb=(H,C,W,G,B), and Rb, the symbol for the state of the right bank is Rb=(0,0,0,0,0)

With these notations, we can deduce that the final state of the problem can be described by the pair of 5-dimensional vectors (Lb, Rb), where Lb=(0,0,0,0,0), Rb=(H,C,W,G,B)

Transactional operators. Transactional operators of different states of the problem, shall be represented by different configurations of boat departures from one bank of the river (the one on which the boat can be found) to the other one. As an example, these operators can be represented by a list of two elements, namely, the first element is the human, and the second is one of the other elements involved in the problem.

From the conditions of the problem, we can deduce that the transactional operators are as follows: B(H,C); B(H,W); B(H,G); B(H,0), where the value 0 represents the lack of the second element in the boat.

By trying to apply these operators, we observe that in the beginning, only the operator B(H,G) can be applied.

The problem solving algorithm cab be:

B(H, G), Lb=(0, C, W, 0, 0), Rb=(H, 0, 0, G, B);
B(H, 0), Ms=(H, C, W, 0, B), Md=(0, 0,0, G,0);
B(H, L), Ms=(0, C, 0, 0, 0), Md=(H, 0, W, G, B);
B(H, C), Ms=(H, C, 0, G, B), Md=(0, 0, W, 0, 0);
B(H, V), Ms=(0, 0, 0, G, 0), Md=(H, V, W, 0, B);
B(H, 0), Ms=(H, 0, 0, G, B), Md=(0, V, W, 0, 0);
B(H, C), Ms=(0, 0,0, 0, 0), Md=(H, V, W, G, B).

By carefully analyzing the solution to this problem, we observe that the order of applying operators is imposed by the compatibility of the elements involved in the problem.

More precisely, we observe the fact that between the elements involved in the problem there is only one compatibility: (C,W), any other two combinations being incompatible.

Consequently, the solution presented by us, can be applied to a larger class of problems, namely for problems with three elements in which only two are compatible.

Taking into consideration the problem of missionaries and cannibals, we can offer a solution.

The problem of cannibals and missionaries is as follows: three missionaries and three cannibals are on the left bank of a river, where there is a boat which allows the six to cross to the other bank. But there is only room for two people at a time on the boat. What solution will the missionaries find, knowing that if on the same bank there are more cannibals then missionaries, the missionaries will be eaten by the cannibals?

The problem will not be solved directly, but we shall try to apply the method of mathematical induction.

If we have one missionary and one cannibal, then there is no problem, they can both cross in the same boat.

If there are two missionaries and two cannibals, then the problem becomes slightly more difficult and belongs to the category of transformation problems.

We shall introduce the following notations:

M, to represent a missionary;
C, to represent a cannibal;
B, to represent the boat.

To begin with, we shall emphasize the space of the problem states. Thus, we shall represent each state of situation in which we may find ourselves throughout solving the problem, by means of two variables, each of them materializing the state of the two banks of the river. This shall be accomplished by means of two lists, one for each bank, each list made up of three elements: the first element is the number of missionaries, the second element is the number of cannibals, and the third element will represent the presence or absence of the boat on the bank.

Thus, the initial state of the left bank can be represented by the combination Lb=(2M,2C,B), and a combination Rb=(0,0,0) as the initial state of the right bank. The final state shall be represented by the lists Lb=(0,0,0) or even Lb=(, ,) for the left bank and Rb=(2M,2C,B) for the right bank.

By value 0, we expressed the fact that the element in that specific position is missing on the respective bank.

Transaction operators in this problem might be B(M,C), B(M,), B(M,M), B(C,C), B(C,)

Taking into consideration the conditions imposed in solving the problem, it can be represented as follows:

B(M, C), Lb=(M, C,), Rb=(M, C, B);
B(M,), Lb =(2M, C, B), Rb =(, C,);
B(M, M), Lb =(, C,), Rb =(2M, C, B);
B(M,), Lb =(M, C, B), Rb =(M, C, B);
B(M, C), Lb =(, ,), Rb =(2M, 2C, B).

Assuming that there are now 3 missionaries and 3 cannibals, the problem becomes even more complicated

Going through the same procedure as in the problem of the 2 missionaries and 2 cannibals, we shall emphasize the space of the problem states. Thus, a combination Lb=(3M,3C,B) might represent the initial state of one bank of the river, say the left one, while the combination Rb=(0,0,0) represents the initial state of the other bank, the right one. The final state shall be represented by lists Lb=(0,0,0) for the left bank and Rb=(3M,3C,B) for the right bank.

Transactional operators. Transactional operators of different states of the problem shall be represented by different configurations of boat departure, from one bank of the river (the one on which the boat is found) to the other one. As an example, these operators can be represented by a list with two elements, namely the first element represents the number of missionaries in the boat, the second the number of cannibals in the boat. According to the current state, it is obvious that the list representing an applicable operator can have one of the following values: B(2,0), B(1,1), B(0,1), B(0,2), or B(1,0).

Solving algorithm. In the initial state of the problem, when on a bank there are the three missionaries, the cannibals and the boat, and on the other bank there is nothing, the state variables Lb, respectively Rb, describing the two banks of the river can be represented R1=(3,3, YES) and R2=(0,0,NO). The state variable R2 may no be represented, since it can be deduced by calculus, starting with R1.

Starting from this theory, any operator can be applied, only that operator B(0,2) is dangerous for the missionaries, so it can never be used.

If we apply the transformation operator B(1,1), then the state variables will have values R1=(2,2, NO) and R2=(1,1,Yes) which will describe another intermediary state of the space of the problem states.

This stage in the problem solving is, at least up to present stage of development of science and technique, and maybe for a long time to come, very difficult to automate, and solving it requires human intervention. On the contrary, the stage of solution searching, by going through a graph of states, is completely automated.

A possible solution may be represented as follows:

B(1, 1), Lb=(2, 2, 0), Rb =(1, 1, 1);
B(1, 0), Lb =(3, 2, 1), Rb =(0, 1, 0);
B(0, 2), Lb =(3, 0, 0), Rb =(0, 3, 1);
B(0, 1), Lb =(3, 1, 1), Rb =(0, 2, 0);
B(2, 0), Lb =(1, 1, 0), Rb =(2, 2, 1);
B(1, 1), Lb =(2, 2, 1), Rb =(1, 1, 0);
B(2, 0), Lb =(0, 2, 0), Rb =(3, 1, 1);
B(0, 1), Lb =(0, 3, 1), Rb =(3, 0, 0);
B(0, 2), Lb =(0, 1, 0), Rb =(3, 2, 1);
B(0, 1), Lb =(0, 2, 1), Rb =(3, 1, 0);
B (0, 2), Lb =(0, 0, 0), Rb =(3, 3, 0).

It can be noticed that there are a lot of ways leading to the solution. Despite this, our solution is not unique, other solutions can be found.

The effectiveness of a search algorithm is measured by the rapidity of finding the solution to the problem, in other words, its capacity to make judicious choices concerning the choice of the operator that is to be allied to each problem state in the solving space.

3. Conclusions

The problems previously discussed are certainly problems that can be seen as a game, or micro-world. Consequently, such a problem is much simpler than real every-day problems. However, this kind of problems represents a core, from which techniques that are nowadays used in solving complex problems have been developed.

Indeed, these examples prove that representing a problem by using concepts of space of problem state and transaction operators, as well as solving it in a process of looking for a way leading from the initial state to the goal (final state), facilitated finding the solution, that is solving the problem in a quasi-natural manner.

In addition to this, we can observe the fact that restriction on element compatibility allowed us to ignore possible alternatives and reduce the search space, at the same time complicating the check test, since with any new state we have to check that the restriction is observed.

A lot of problems in every-day life have this kind of restriction to observe. In many cases we can speak of problems of satisfying restriction, for which search techniques in the state space are applicable, but for solving them we need a mechanism that would allow us to manipulate restrictions connected to each and every state.

REFERENCES

1. Debreu, G. — "The mathematization of economic theory", American Economic Review, 81, pp.1-731, 1991
2. Giurgiteanu N., Buligiu I. — Formalizarea şi rezolvarea problemelor, Timisoara Journal of Economics (forthcoming), 2010
3. Newell A., Simon H. A. — "Human problem solving, Englewood Cliffs", N. J., Erlbaum., 1972
4. Rui Da Silva Neves — "Psychologie cognitive" , Edition Armand Collin, collection Psychologie, 2000
5. Saloner, G. — "Modeling, game theory, and strategic management", Strategic Management Journal, 12, pp.119-136., 1991