# ECONOMIC EFFICIENCY BY DYNAMIC MODELS. THE EVOLUTION OF THE FIRM'S CAPITAL

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**Abstract:** The objective in this paper is to improve the existing evidence regarding the role of dynamic models plays in economic efficiency. In the research to shape the company's activity many types of approach have been crystallized. And so there exist models in which the company is considered a homogenous ensemble in report with its environment for work, the wagers being caught in ensemble as a production-work factor. From this perspective the company follows only to maximize the profit on short term. The changes occurred in the exogenous economic factors of the company have a decisive impact over its evolution.

In this study we approach the development dynamics of a company by a model of economy dynamic. We used the system dynamic method to build a dynamic model with which to determine the evolution of the firm's capital and the workforce. A practical approach is calculated.

**JEL classification: C61, E27, L25**

**Key words: economic efficiency, stability, equilibrium, evolution in time.**

## 1. INTRODUCTION

The economic performances at a microeconomic or macroeconomic level or belonging to some economic wider spaces, regional or worldwide are situated in the universe of some social-politic, technique and economic conditions that are numerous and very complex, and which differentiate from one group of countries to another, from a country to another, and of course, from one company to another. The logic request, which is took from this fact, is investigating the management systems implicated, so that there is assured an informational database superior for all the types of management as a knowledge of leading [4].

The company's management is a synthesis economical discipline with a multidisciplinary character of which the objective is to study the management processes and relations in their frame, concerning discovering the principles and laws that govern those framing new systems, methods, techniques and modalities of leading that may assure the efficiency growth. The management process assures the potential of the execution process and contains the provisory phase, the operationalization phase and the evaluation phase and interpreting the results.

As the economic process and phenomenon's complexity grows, also the informational-decisional with which the decisional factors confront with from the companies frame, the theoretic and practical preoccupations for finding some solutions to perfect and raise the management activity efficiency.

This way the empiric management developed exclusively on the basis of judgment and the managers experience should be reduced and putt more accent over growing the moderation of the scientific instrument of management putt at the decedents disposal, as well as an intensification of the creative device for the adaptation of this instrument in the concrete conditions of every situation.

Both the researchers and the managers have started to see with more and more elaborated ways, that the management processes are conditioned concomitant by economic, social, psychosocial, technological, physical-geographical factors, preparing this way the treatment for the management by the prism of implicated elements ensemble.

An investment is done according to a project done considering the development strategy of the company. The strategy prefigures the way in which it intentions to engage in the competitive fight, using its organizational resources in the purpose of obtaining and strengthening its competitive advantages. An investment strategy consists in significantly enlarging the business volume, fact that attracts usually important sale enlargement, and so, the profits also [5].

The necessity of optimizing the decisions, stabilizing the successions of moves, operations, processes, combinations or favorable conditions for the activity planned by the manager, growing the efficiency of the company management are some of the reasons for which it is appealed to the simulation technique.

The key for surviving and growing is constituted by the capacity of a firm to adapt it's strategies with the environment permanently in change. This requires the management to anticipate correctly the future events. The anticipation is realized by the planning departments which develop long term previsions of the most important factors that affect the products market. Specialized firms realize long term previsions for different components of the macroeconomic medium like, for example, the economy, the population, natural resources and technology. The experts opinion, exploiting the trend, correlating the trend, econometric modulation, cross-impact analysis, request-hazard prevision also multiple scenarios are only some methodologies of realizing the macroeconomic previsions.

Previsions are made on the base of the dates and models and depend very much of these data. Economic models are powerfully influenced by the period in which the data was gathered and it's size. The modulation consists in the fact that constructing a representation with a variable degree of fidelity of the real world or a component part of it. Understanding this phenomenon or its segment of reality took in consideration, knowing by detail also the action over the phenomenon that is analyzed motivates the ration for taking this kind of representations. The most used language is the mathematical language. Using the mathematical modulation helps the substantiation of a decision in the efficiency conditions, offering the possibility to think better and faster without denaturizing the reality. Mathematical formulating is not the only kind of abstracting form, but "it is the superior step of scientific abstract, which approaches us to the frontiers of certitude insofar as the premises are not falsified" after the T. Shattles firm.

Starting from the idea that any model is based on real parameters and data, it becomes necessary the fact that for obtaining trustful dates to permit a confinable representation of the reality by model. Therefore, when necessary, the cyclic or periodic aspect of the phenomenon studied is identified, implicitly the time horizon which is referred to.

Mathematising the economical disciplines has driven to crystallizing the micro and macroeconomic analysis, indifferent of the branch particularities, aborting common problems of any economy sector. The microeconomic is concerned as an isolated element of a bigger system, but not as something existing for itself. Economic systems are hierarchical synergetic systems, different economic dynamics corresponding to the level to which it is studied the economic phenomenon. The microeconomic models constitute the cores or "bricks" of which are composed macroeconomic models. Estimating the parameters of macroeconomic models in dependence of microeconomic characteristics is, actually, the main purpose of economic mathematics, like also other domains of quantitative economical domains, of high utility by the economic plans and politics point of view [8].

By economic system it is understood a system (primary notion) characterized by economic sizes. The economy is an analytic study which concerns of the existing relations or it can be supposed to exist between direct measurable sizes. The prices, mortgages, incomes, costs, quantity of sold product or bought by the market and quantities of factors of protection used by an enterprise are only some examples of variable sizes used in economy. Some of these sizes can be measured in natural or physical units, others only in money or other value units. Important for us here is that they are measured in some ordinary units. So there is no doubt that the mathematical methods can be applied in economy and that the economical relations may be expressed with the help of some mathematical functions although sometimes economy conditions of the problem impose certain limits concerning the form of the functions, systems, etc.

Using econometric modulation researchers have realized equation systems which describe condition systems. Econometric models which contain more than 300 equations, for example, are used for premising the changes in American economy.

The activities in the economic space, for the individual economic agents or for an ensemble of economic agents which constitute in a bough or under of activity, or for their totality, at the level of national economic space, there can be analyzed, followed and optimized and by the application of some new modern methods, in a conception of aborting mathematics, in which the mathematical instrument plays a very important role. This kind of instrument is the theory of dynamic systems.

## 2. DYNAMICAL MODELING OF BUSINESS ACTIVITIES

Schumpeter's theory (1934) of the entrepreneur is a part of a theory of the economic development. According Schumpeter, entrepreneurs are economic agents whose functions are the "carrying out of the new combinations", the creative destruction of equilibria, thereby preparing the ground for a superior state of equilibrium.

An essential aspect of Schumpeter competition is that are winners and losers and the process is one of continuing disequilibrium. Firms facing the same market signals respond differently and more so if the signals are relatively novel. In order to get entrepreneurial profits, firms have to solve many technical, economical and psychological problems. Firms participate innovative competition through product

development, price or cost competition through process development, and marketing competition through sales or market promotion. Failures in any competitive field may affect incomes of the firms.

The works by Morishima(1964,1969), Hicks(1965), Leontief(1949), Sraffa(1960), Robinson(1953), von Neumann(1937) and others have played a significant role in the development of modern economic theory.

In the '30s, Jan Tinberger began his research and elaborated the first macroeconomic methods with more equations. Their modern prototype, crystallized, is the Klein-Goldberger (1955) model which unites different economic variables – income, consume, taxes, spends, investments – by a round of macroeconomic variables – for example, functions or request.

Dynamical modeling was elaborated by J. Forrester in the period of the '60s and is based on the fact that the functioning of a system is represented by the knowledge of the interactions between the information fluxes, commands, human resources and material resources etc. A dynamic model surprises the behavior or complex systems showing how their structure determinates their trajectory, respectively their behavior in time.

In the economical dynamic there are met hundreds of mathematical models. They contain numerous parameters, this implies a high difficult apply and concerning the mathematics is reclaims using some topological methods of the singularities theory (catastrophes and bifurcations). While the parameters effects described by the global diagram of dynamic bifurcation is very different in different zones of the parameters space, finding the zones of structural stabilization and of inexistence of this stability (of bifurcation) is vital for the management of an economy.

The appearance of the theory of nonlinear dynamics has permitted the understanding and development of some processes and methods to approach us more to the phenomenon of reality. Developing the singularities theory and the bifurcation theory has completed the group of ways we dispose of for analyzing and representing more and more complex dynamics, giving the possibility of analyzing some systems which were hard, if not impossible to talk about by traditional methods. The study over nonlinear dynamics is of maximum interest because the economic systems are by excellence nonlinear systems. Many of these contain multiple discontinues and incorporate an intern instability being permanently submissive to the actions of the shocks, and intern and extern perturbations.

For low values of some parameters there can be produced extremely high changes of variables, so there can be produced bifurcations to subscribe the considered system for other trajectories.

The bifurcation theory has the advantage of a mathematical device very good elaborated, studying the existence and the stability of balance solutions, because a solution in instable balance cannot be observed in reality [9].

Because of the complexity of real processes, in the construction of methods it must be adopted a certain limit of detailing, restraining the essential elements and the main dependences between them. After that, the model must always be a simplified representation of the reality to permit actions, based on rationality, over the modulated process.

Dynamic modulations follow distinguishing of temporary relations. The model operates with the events and settings which express the value of an attribute in which the events apparition is identified. With the help of data structures transition of states

diagrams are built, these indicate the entire operations specific to every type of object and class corresponding.

Nowadays the necessity of dynamic systems study from micro and macro economy justify, along with the ones in biology, the rationality of development without precedent of the theory of the dynamic systems and its applications.

In economic dynamics, previsions are made on the base of the data from the models and depend very much on this data. In an important part these data, which are conform to the correct data registered in economic development, they are not made public. The economic models are powerfully influenced also by the period in which these data have been gathered and it's size.

Economical processes described by ordinary differential equations are named continuous processes. If the evolution is examined in discrete timing (year, month, trimester, semester, decade, week, day, hour) or by introducing a unity of stimulation, then the process dynamics will be described by discrete methods consisting of equations with infinite differences, equation systems with finite differences or recurrent equations.

The model "good", "stalwart", when the process structure on which is applied does not suffer serious changes at the data variant, so that the dynamic system corresponding is stabile structural For example, if the model foresees an evolution in a certain cyclic, than it's confirmation takes place only if in the system there has not been produced any structural mutations, like a massive intervention of the state which modifies the entire mechanism of reactions of the system. The problem of economic models is a problem of the type of dynamic process to which is applied in consequence of the data type series corresponding to the process.

The evolution of economical processes can be represented in different dynamic models concerning the complexity of the process, when a single main indicator y (t) is distinguished respectively an ensemble of indicators y1 (t) … yn (t) correlated by the equations of the model both between them and with the factorial variables which condition the process u1 (t) ... um (t). The indicators are the unknown functions and represent the variables of state of the associated dynamic system. Treating the economic process as a system distinguishes the factors u1 (t) ... um (t) as entrances and, the indicators $\dot{y}_1(t)...\dot{y}_n(t),\ y_1(t)...y_n(t)$, as variables of state of the system, outputs.

In a mathematical point of view, the factors are data.

We must remark that the economic process and phenomenon modulation method is, in present, a method of reference for the theory and practice of mathematic modulation. The model is built as an isoform representation of reality and offers an intuitive image but also rigorously to it in the sense of the logic structure of the studied phenomenon. This way, it is facilitated the discovery of some links and bindings which, practically, in other ways, would be impossible or very hard to find. The explanation of some evolutions is built on the results of such estimations.

Because of the complexity of real processes, in the constituting of models a certain detail limit must be adopted, restraining the essential elements and the principals depending on them. That is why the model must be always a simplified representation of the reality which is to permit actions, based on rationality, over the modeled process.

This paper builds a dynamic model to examine interdependence between firm's capital and firm's workforce. This model is influenced by the neoclassical trade theory with capital accumulation. Since the publication of Oniki and Uzawa's paper on theory

of trade and economic growth (Oniki and Uzawa, 1965), various trade models with endogenous capital have been proposed (e.g., Deardorff, 1973, Ruffin, 1979, Findlay, 1984, Eaton, 1987, Brecher, etal., 2002, Sorger, 2003).

Our main contribution is that we investigate various nonlinear dynamic phenomena such as bifurcation and economic cycles. We emphasize that oscillations and structural changes are not rare but universal in a progressive economy. No economic system can be stabilized if change is permitted.

## 3. RESULTS AND DISCUSSION

The apparition of the nonlinear dynamics had permitted the understanding and the development of some processes and methods that draw near the phenomenon to reality. The development of the theory of singularities and the theory of bifurcations had completed the multitude of instruments of analyzing and represents dynamics more and more complex giving the possibility of analyzing systems that were hard even impossible to use them from the traditional point of view. The study of nonlinear dynamics is very important for us because the economical systems are defined as nonlinear. Much of them contains multiple discontinuities and incorporates an inherent instability being permanently submitted to the actions of shocks and external and internal perturbations.

### 3.1. THE MATHEMATICAL MODEL

The continuing economical development is the result of combined contribution of two factors of productions: capital (K) and the force of work (L). These two factors are the initial variables of the model considered by us. We will observe the variation in time of capital and force of work beginning from initial important data corresponding to some dots from the space of parameters.

Let $K_t$ be the capital of a firm at the time $t$ and let $L_t$ be the number of workers. Then the production force reads $y_t = F(K_t, L_t)$. The dynamics of the capital depends on the politics of development of the firm involving the net profit $\pi_t$, the dividends covering by the shareholders $\delta_t$ (where $\delta_t \pi_t$ represents the dividends and $(1-\delta_t)\pi_t$ are the remaining investments), the capital depreciation by a coefficient $\mu_t$ and the income obtained by liquidation of the damped actives at the revenue costs $\lambda_t$. Let $\gamma_t$ be the rate of change of the capital, such that $\pi_t = \gamma_t y_t$. Then,

$$\dot{K}(t) = \gamma_t(1-\delta_t)F(K_t, L_t) - \mu_t(1-\lambda_t)K$$

In [8] we will suppose also that the variation of the force of work is

$$\dot{L}(t) = \alpha_1 K + \alpha_2 L - \alpha_0$$

and the company is characterized by a function of production as the from of Cobb-Douglas, $y_t = AK^{\alpha}L^{\beta}$, for whom we distinguish three cases $\alpha + \beta < 1$ – production with physical decrease effective power, $\alpha + \beta = 1$ - production with

physical constant effective power, $\alpha+\beta>1$ - production with physical growing effective power.

In this system K and L : R→R are unknown functions which depends by the independent variable t (time). If $y_t = AK^{\alpha}L^{\beta}$ and the production has an increasing physical efficiency, i.e. $\alpha+\beta>1$, the above equations become

$$\begin{cases} \dot{x} = cx^2 y + bx \\ \dot{y} = x + \alpha_2 y - 1, \end{cases}$$

where we choose $\alpha=2$, $\beta=1$, $x=\beta_1 k_t$, $y=\beta_2 L_t$, $\beta_1=\alpha_1/\alpha_0$, $\beta_2=1/\alpha_0$ for $\alpha_0\neq 0$, $\alpha_1\neq 0$, $a=A\gamma_t(1-\delta_t)$, $b=-\mu_t(1-\lambda_t)$, $c=a\alpha_0^2/\alpha_1$. In this way the new states functions $x$ and $y$ are proportional to the capital and working force respectively. Thus, in equations remains just three parameters b, c and $\alpha_2$.

Numerical application

In this model we considered a function of production as model Cobb-Douglas with coefficients of elasticity uncomplimentary with growing effective power. If we suppose now the particular case in which the decrease of the capital is realized with medium coefficient $\mu=5\%$ - rate of meeting the expensive of dividends by shareholders of the company is $\delta=50\%$, the rate of net income in business income is $\gamma=10\%$ and the incomes earned by the abolition of the amortized actives has the cost of coming back $\lambda=0{,}8$ (mil.lei/1 mil.lei abolished capital).

If the initial data are: $K_0=2$ mil. lei, $L_0=2$ ( hundred of persons) $Y_0=16$ mil.lei. Than, the constant A becomes $A=\dfrac{Y_0}{K_0^2 L_0}=2$.

Replacing these numeric data in the model we obtain $b=-0{,}01$ $c=0{,}1$. Varying the $\alpha_2$ parameter we will study now the evolution of the functions $x=K$ and $y=L$.

1) $\alpha_2=2$, $b=-0{,}01$ $c=0{,}1$. In this situation it is observed a fast growth of work force while the capital records in the part of the beginning a slow growing for then to rise towards 15 mil.lei. (figure 1)

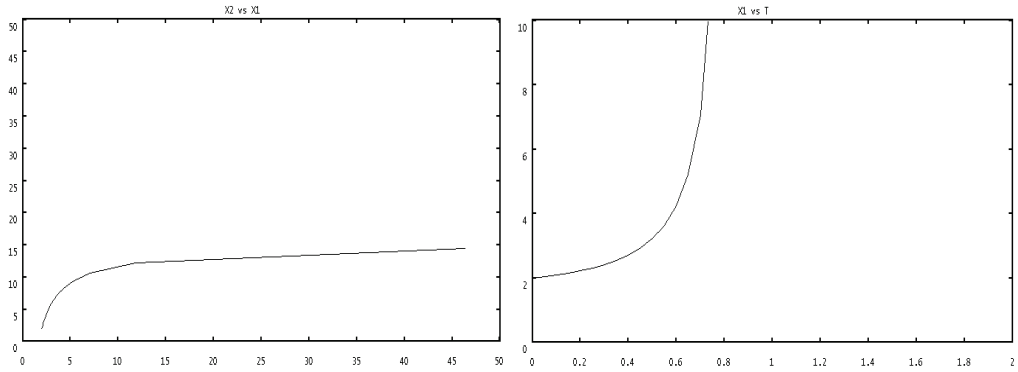


Figure 1. The evolution of: a) capital in time; b) workforce in time

2) $\alpha_2 = 5$, $b = -0{,}01$, $c = 0{,}1$. Here, the work force is also a function that grows in time, the capital has a calm period at the beginning, and after that it goes asymptotic towards 30 mil.lei. (figure 2)

If we choose $\alpha_2 = \alpha_1 = -1$ we will obtain $b = -0{,}01$, $c = -0{,}1$. For keeping the economic significance, we put in this situation $x = K$, $y = -L$ we suppose. $L_0 = -2$.

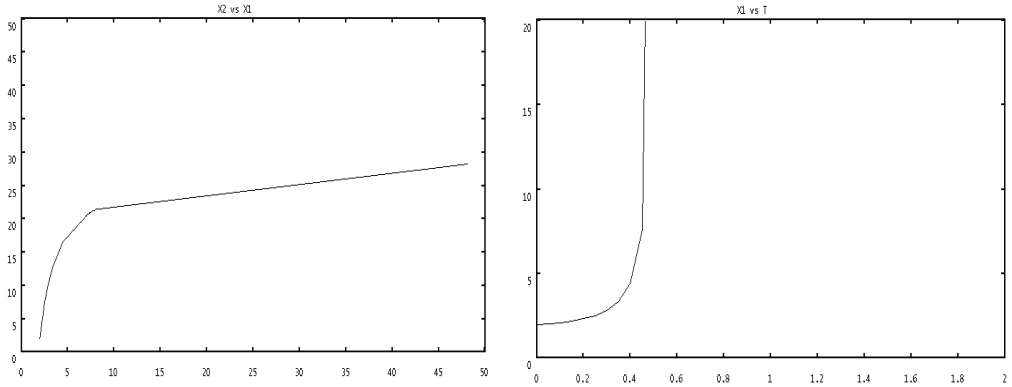


Figure 2. The evolution of: a) capital in time; b) workforce in time

3) $\alpha_2 = 1$, $b = -0{,}01$, $c = -0{,}1$. In this case we observe a slow period of capital growth and after there follows a short period of prosperity. At the same time, the work force records an unimportant growth tending towards zero at the moment when the capital reaches its maximum, afterwards appearing a crisis that leads to the lowering of the capital, and to failure.(figure 3)

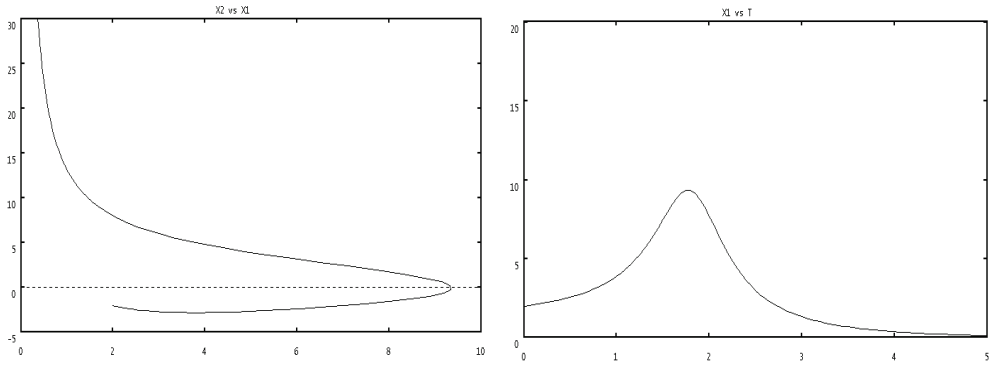


Figure 3. The evolution of: a) capital in time; b) workforce in time

4) $\alpha_2 = b = -0{,}01$ , $c = -0{,}1$ . As it is demonstrated in [6], in this situation the system admits a periodic solution.(figure 4)

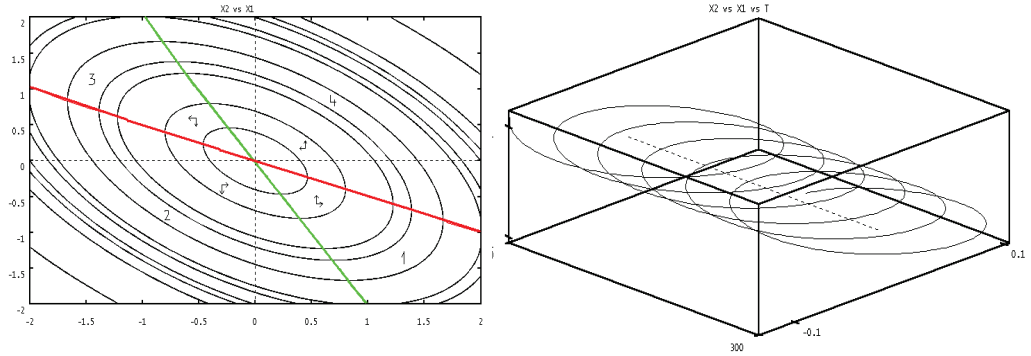


Figure4. a) The evolution of the capital with workforce. b) The periodic solution corresponds an economic cycle.

To this periodic solution corresponds an economic cycle. The main crisis is a sudden, violent, perturbing phenomenon that can be translated by a brutal fall, by the failure of the enterprise and massive dismisses of work force. The crisis is a temporary breakage but sometimes violent and profound on an ascendant evolution of economy, of balance between the production and the need, that follows after the maximum point of growth in a cycle and it is expressed by an investment of the tendency that makes the economy balance from growing to depression. Locating the crisis between the two economic cycles must not lead in any way to associating or mistaking it with the depression period that succeeds it. A clear distinction must be made between the crisis and the depression. The depression is opposed to the violent, unexpected, perturbing character of the crisis, as a slow period of reorganization, to put in order the productive system, by which course there are manifested negative phenomenon like: the lowering of production, salary and the unemployment growth.

The depression is the phase that succeeds the crisis and it is characterized by the economic activity stagnation and even continuous diminishing of production, and the growing unemployment. A reduction of the current production volume takes place, there are adopted drastic measures of reducing the costs, from which departs upgrading the working devices active on the basis of a technical innovation. Rerunning or increasing the economic activity is tightly tied of renewing the fix capital and especially its active parts, which leads to overtaking the lowest point of the respective cycle.

Refreshing the investment process concerning creating new capacities and also renewing the existent ones, supplies the requests of production devices and work force, reducing the unemployment rate. The expansion or advance is characterized by and abundance of credit and a general growth of incomes. In this kind of atmosphere, more and more favorable, the businesses become prosperous, with a consolidation perspective. An investment process is launched; the upgrading of existent production capacities takes place and also creating new ones. The sustained process launched by the growth of consumable goods, is the determinant factor of growth in production and the grade of work force occupation.

### 4. Analyses Conclusion and Prospects for Further Research

Generally, with the activities being more complex, with the need of planning, search for strategies and formal actions grows. The economic domain is a domain in which the uncertain grade and risk is very high and in which the planning plays an important role in trying to reduce this incertitude. In essence, the elaboration of strategies in this domain purposes a clear and systematic structure of the modulations in which the followed objectives can be touched by a judicious allocation of the resources by long or short term.

In conclusion, private economic simulation as a management component has a theoretical-practical end only by using some modern methods, performances, of analysis and quantification, capable to surprise the essence of the economic phenomenon and processes researched, to evaluate their dimensions in reality, their tendencies of evolution in the future, them being influenced by numerous factors, that are in interconditioned realities.

This study is only a starting point of economic dynamics. We are confronted with more difficult analytic problems: economic systems are described by unstable nonlinear dynamic equations of high dimensions with different adjustment speeds. The existence nonlinear theories have significant implications for economic forecasting, methodologies and so on.

## References


1. Arrowsmith, D. K., Place, C.M. An Introduction to Dynamical Systems, Cambridge University Press, 1990
2. Beltrami, E. Mathematics for Dynamic Modeling. Academic Press, New York, 1990
3. Braun, M. Differential Equations and Their Aplications. 3-rd ed., Springer, New York, 1983
4. Everett. E., Adam J.R. Ronald J. Ebert Production and Operation Management, Teora Press, Bucharest, pag. 38-43, 2001
5. Firică O. Fundamental models for decision in organizational management, Matrix Rom Press, Bucharest (in romanian), 2002
6. Gandolfo, G. Economic dynamics, Springer, Berlin, 1996
7. Oprescu G. Mathematics for economists, "Fundatia Romania de Maine" Publishing House, Bucuresti, (in romanian), 1996
8. Popescu I., Ungureanu L. The paradigm of economic complexity", Expert, Bucharest 2006 (in romanian).
9. Ungureanu L. Structural Stability and Bifurcation in two Models of Economic Dynamics, Piteşti University Press, Piteşti (in romanian), 2004

10. Ungureanu L., Matei V. Nonlinear problems in economic development, Journal of Applied Economic Sciences, Volume 2, Issue 1(2); 2007
11. Ungureanu L., Matei V. Advances in Decision Analysis. Efficient Methods in Finance, Journal of Applied Economic Sciences, vol. 3, Issue 4(6), 2008
12. Zhang W-B., A multi-region economic growth model with migration, housing and regional amenity, Analele Stiintifice ale Universitatii "Alexandru Ioan Cuza" din Iasi - Stiinte Economice, vol. 55, pages 322-350, November, 2008